PROBING NEUTRINO OSCILLATIONS AROUND BLACK HOLES

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Abstract. *We investigate how dark matter in a phantom-field background (DMPF) surrounding a Schwarzschild black hole modifies neutrino flavor oscillations within a two-flavor model. For radially propagating neutrinos the gravitational contributions cancel, so the oscillation phase grows with distance and mass-squared splitting exactly as in flat spacetime.*

1. Introduction

Black holes offer a natural laboratory for testing general relativity under extreme conditions, as confirmed by gravitational waves [1] and horizon-scale images [2,3]. Because dark matter and dark energy dominate the universe [4,5], this paper uses a two-flavor model to examine how a dark-matter-in-a-phantom-field (DMPF) background [6-8] around a static black hole affects neutrino flavor oscillations.

2. Spacetime with Dark Matter in a Phantom Field

We model the dark component by a scalar (phantom) field minimally coupled to gravity through the Einstein–Hilbert action; variation with respect to the metric yields the field equations. The resulting static, spherically symmetric solution [7,8] can be written as

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\theta^2 + r^2\sin^2\theta\,d\varphi^2$$

with the metric function

$$f(r) = 1 - \frac{2M}{r} + \frac{a}{r}\ln\left(\frac{r}{|a|}\right)$$

where M is the black-hole mass and the logarithmic term, proportional to the dark-sector parameter a, encodes the dark-matter and phantom-field contribution.

3. Oscillation Phase in Flat and Curved Spacetime

Neutrino flavor states are superpositions of mass eigenstates through the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix [9,10],

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U^*_{\alpha i}\,|\nu_i\rangle$$

In flat spacetime, the plane-wave phase difference between two mass eigenstates is

$$\Delta\Phi_{ij} \approx \frac{\Delta m^2_{ij}}{2E_0}\,|x_D - x_S|$$

with $\Delta m^2_{ij} = m_i^2 - m_j^2$ and E_0 the neutrino energy at infinity. To generalize to curved spacetime we use the covariant phase of Stodolsky [11],

$$\Phi_k = \int_S^D p^{(k)}_\mu\,dx^\mu\,, \qquad p^{(k)}_\mu = m_k\,g_{\mu\nu}\frac{dx^\nu}{ds}$$

evaluated along the null trajectory of a massless reference particle. For purely radial propagation ($J_k = 0$), the mass-shell condition together with the ultra-relativistic limit $m_k \ll E_k$ reduces the phase to

$$\Phi_k \approx \pm \frac{m_k^2}{2E_0} (r_D - r_S)$$

so the gravitational terms cancel and the result coincides with the flat-space expression. The situation differs for non-radial, gravitationally lensed trajectories characterized by an impact parameter b. Carrying out the phase integral over the bent path in the weak-field limit, and taking $b \ll r_s, r_ə$, gives

$$\Phi_k \approx \frac{m_k^2}{2E_0}(r_S + r_D)\left[1 - \frac{b^2}{2r_S r_D} + \frac{2M}{r_S + r_D} + \frac{a}{r_S + r_D}\ln\left(\frac{r_S r_D}{M^2}\right)\right]$$

The leading term is proportional to the total path length; the finite impact parameter, the lens mass M, and the dark-sector parameter a all enter as corrections, the last through the characteristic logarithmic term $\ln(r_S r_D/M^2)$. Thus DMPF imprints itself on the oscillation phase only when the neutrino trajectory is deflected.

4. Oscillation Probability and the Two-Flavor Model

When gravitational lensing produces several classical paths, the state at the detector is a coherent sum over both mass eigenstates and trajectories, and the flavor-transition probability takes the form

$$P_{\alpha\beta} = |N|^2 \sum_{i,j} U_{\beta i} U^*_{\beta j} U_{\alpha j} U^*_{\alpha i} \sum_{p,q} e^{-i\Delta\Phi_{ij}^{pq}}$$

where the double sum combines the usual flavor interference with interference between paths. The lensing phase difference splits into mass- and geometry-dependent pieces, $\Delta\Phi^{pq}_{ij} = A_{pq}\,\Delta m^2_{ij} + B_{ij}\,\Delta b^2_{pq}$, with the dark-sector parameter entering the coefficient A_{pq} through the same logarithmic term.

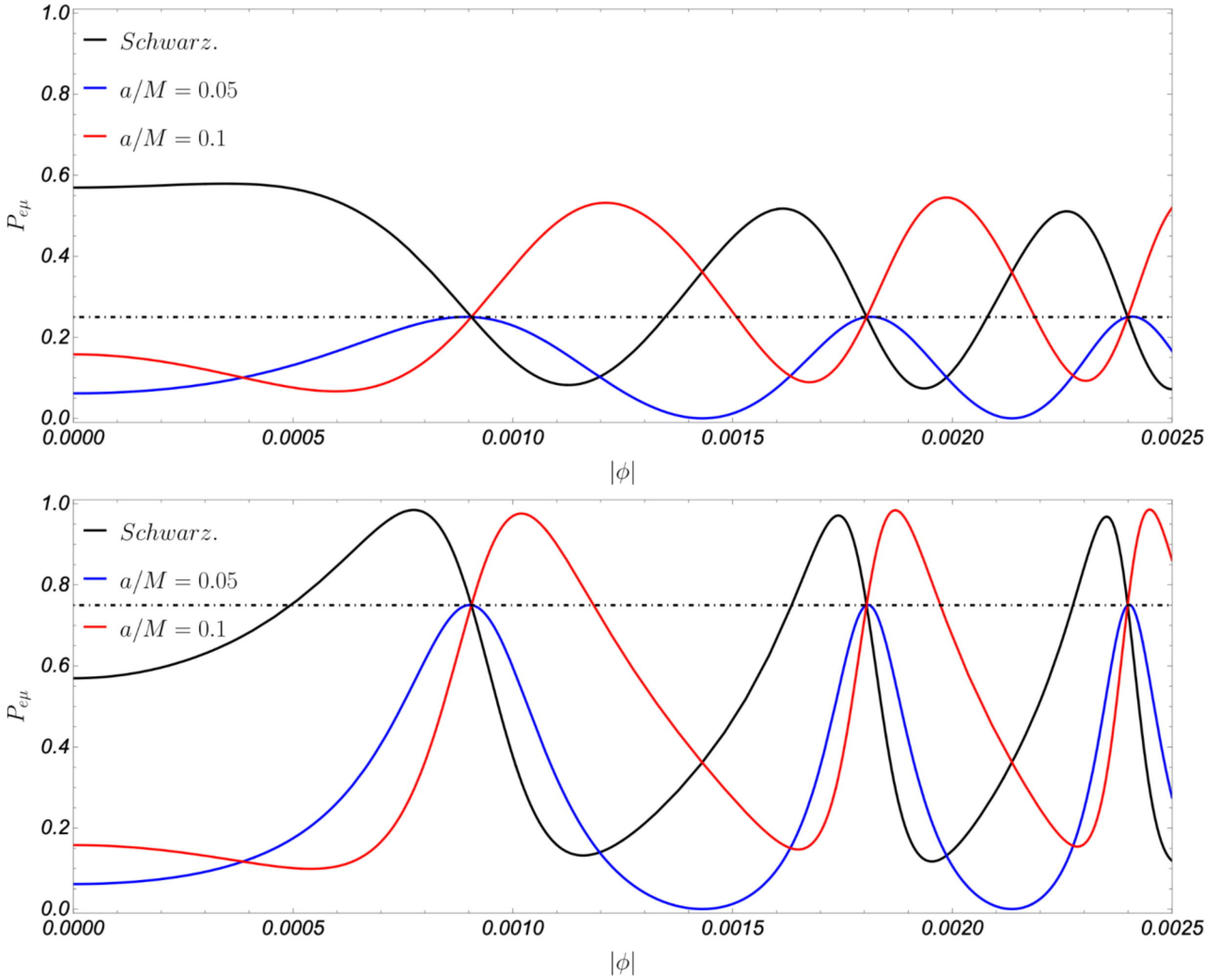

Figure 1. *Transition probability $P(\nu_e \to \nu_\mu)$ versus azimuthal angle φ for the Schwarzschild limit (a = 0) and the DMPF cases a/M = 0.05 and 0.1, for the normal (top) and inverted (bottom) mass hierarchies. Fixed: $\alpha = \pi/6$, $M = M_\odot$, $\Delta m^2 = 10^{-3}$ eV^2.*

Solving the weak-lensing deflection condition yields two positive impact parameters b_1, b_2 for each source angle. Specializing to a Sun–Earth configuration ($r_D = 10^5$ km, $r_S = 10^5$ r_D, E_0 = 10 MeV, $|\Delta m^2| = 10^{-3}$ eV^2, $\alpha = \pi/6$, $M = M_\odot$) and reducing the three-flavor problem to an effective two-flavor model, we compute $P(\nu_e \to \nu_\mu)$ as a function of the azimuthal angle φ.

The transition probability is sensitive both to the dark-sector parameter a/M and to the mass hierarchy (Fig. 1). We further find a degeneracy between the lens mass M and a/M: different combinations of the two yield the same probability, with the inverted hierarchy requiring a slightly larger a/M than the normal hierarchy to reproduce a given $P_{e\mu}$, a difference that grows with the lens mass.

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