TEACHING GEOMETRY IN ACADEMIC LYCEUMS THROUGH LESSON TECHNOLOGICAL MAPS AND DIGITAL TOOLS

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Abstract. *The article proposes a methodological model for teaching geometry in academic lyceums that combines lesson technological maps with digital tools. The model has five components: target, content, procedural, technological (digital) and evaluative. Its procedural core is a two-page technological map that fixes five stages of an 80-minute lesson, the teacher's and students' actions at each stage, the methods used and the time allotted. The map is built in three variants — for theoretical, practical and assessment lessons — and each stage is linked to a concrete digital tool: dynamic geometry software (GeoGebra) for constructions and spatial figures, online quizzes for formative assessment, and shared documents for homework. The model was applied to the full two-year geometry curriculum of an academic lyceum (16 sections, 144 lessons, 288 hours): 54 theoretical, 74 practical and 16 assessment lessons received a map, and a bank of worked examples and graded problems with verified answers was compiled. The study is theoretical and methodological; it describes the design principles, the structure of the maps and the conditions of their use, and outlines a pedagogical experiment for testing the model's effect on students' geometric reasoning.*

Keywords: *geometry teaching; academic lyceum; lesson technological map; instructional design; dynamic geometry software; GeoGebra; formative assessment; spatial reasoning.*

1. Introduction

Geometry is the part of school mathematics where students first meet proof, construction and spatial reasoning, and it is also the part where many of them fall behind. Uzbekistan's first participation in PISA in 2022 made the scale of the problem visible: the national mean in mathematics was 364 points against an average of 472 for participating countries [21], and almost no students reached the top proficiency levels 5 and 6 [14]. The state has named mathematics education a priority: Presidential Resolution PQ-4708 of 7 May 2020 set the improvement of mathematics teaching in general secondary and secondary specialized institutions as a key direction and called for online platforms and better assessment [15].

Academic lyceums sit at the point where these goals meet the classroom. In two years their students complete a geometry course of 288 hours that runs from the axioms of planimetry to combinations of solids, and most of them then sit university entrance tests. The course is dense: a typical theoretical lesson in the lyceum curriculum covers what a general school spreads over several weeks, for example the sine, cosine and area theorems of a triangle in one 80-minute session [7]. Under this pressure a lesson easily turns into a lecture followed by unstructured problem solving, with little time left for construction, visualization or feedback.

Two instruments are widely used to bring structure and visualization into such lessons, but they are rarely combined. The first is the lesson technological map, a document from the pedagogical technology tradition that fixes the stages of a lesson, the actions of teacher and student and the expected result at each stage [2; 16; 18]. The second is dynamic geometry software, which lets students construct, drag and measure figures and thus see invariants that a

static drawing hides [6; 10]. A map without digital tools stays a planning formality; digital tools without a map are used occasionally and without a clear didactic place.

The aim of this article is to propose and describe a methodological model in which the technological map serves as the organizing frame of a geometry lesson and every stage of the map is tied to a specific digital tool and a specific form of feedback. The objectives are: (1) to formulate the theoretical basis of the model; (2) to describe its components and the structure of the map for three lesson types; (3) to show the model's application to the complete two-year geometry curriculum of an academic lyceum; (4) to outline a design for its experimental testing.

2. Literature review

Pedagogical technology and the technological map. In the post-Soviet didactic tradition a pedagogical technology is understood as a reproducible sequence of steps that guarantees a planned result [2; 16]. V. P. Bespalko stressed that a technology must start from goals stated so precisely that their achievement can be checked [2]. G. K. Selevko described the technological map as the practical form of such a technology: a table that sets out the stages of a lesson, the operations at each stage and the means of control [16]. In Uzbekistan the approach was developed by O'. Q. Tolipov and M. Usmonboyeva, who linked the map to interactive methods such as brainstorming, cluster and “Insert” [18]. Close ideas exist in Western instructional design: backward design starts from the desired understanding and the evidence of it [20], and constructive alignment requires that outcomes, activities and assessment point to the same verbs [3]. Both rely on a hierarchy of cognitive outcomes, for which the revised Bloom taxonomy is the common reference [1].

Learning geometry. The van Hiele theory describes geometric thinking as a sequence of levels, from recognizing shapes to formal deduction, and holds that instruction must move students through these levels in order [19]. R. Duval showed that a geometric figure can be read perceptually, sequentially (as a construction), discursively and operatively, and that students who see only the perceptual reading fail to use the figure in proof [5]. These findings explain why a lecture-and-exercise lesson, in which figures are drawn once on the board, often fails at the lyceum level, where deduction and spatial figures dominate.

Dynamic geometry. Dynamic geometry environments let students construct a figure by its defining properties and then drag it: what stays invariant is a theorem, what changes is a special case. C. Laborde showed that well-designed tasks in Cabri turn dragging into a tool for conjecture and proof [10]. K. Jones found that work in a dynamic environment helped students move from describing figures to explaining them [9]. GeoGebra joined dynamic geometry with algebra and coordinates in one free tool [6], and its 3D view brought the same approach to solid geometry. A review of the field by Sinclair and colleagues confirmed the value of dynamic geometry for conjecturing, but also noted that its effect depends on task design and teacher orchestration [17]. P. Drijvers reached a similar conclusion for digital technology in mathematics in general: it works when its role in the lesson is didactically defined [4].

Teacher knowledge. The TPACK framework describes the knowledge a teacher needs to combine technology, pedagogy and content [11]. For geometry this means knowing not only how to use GeoGebra, but at which stage of which lesson a construction, a drag or a quick quiz serves the goal. The author's earlier work on task design in mathematics [13] and on differentiated teaching of “Algebra and Foundations of Analysis” in the lyceum [12] approached this question from the side of tasks and student groups.

Research gap. The literature offers a planning instrument (the technological map) and a visualization instrument (dynamic geometry), and it states that the second works only inside a clear lesson design. What is missing is a model that specifies, for a complete lyceum geometry course, which digital tool is used at which stage of which type of lesson. This article addresses that gap.

3. Methodology: the model

The study uses theoretical methods: analysis of the pedagogical, psychological and methodological literature, analysis of the lyceum geometry curriculum and its calendar plan, modelling, and design of teaching materials. The result of modelling is a five-component model (Figure 1).

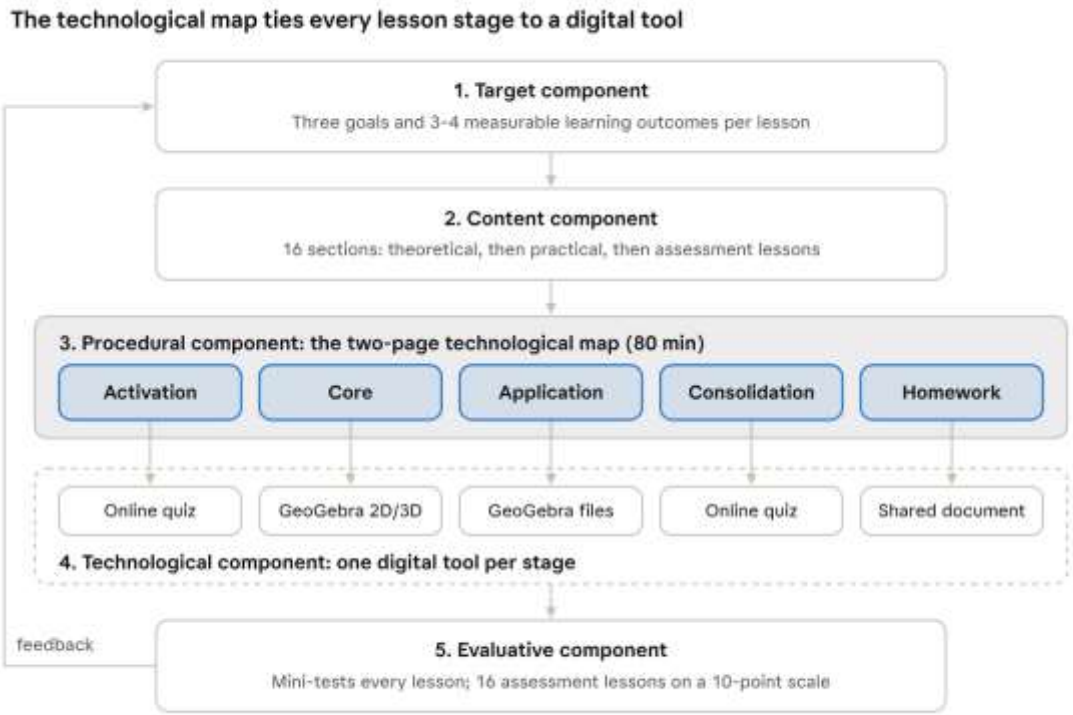

Figure 1. Methodological model of teaching geometry with technological maps and digital tools

Figure 1. The five components of the model. Assessment results return to the target component and adjust the plan of the next section.

1. **Target component.** Each lesson has three goals — educational, upbringing and developmental — and three to four learning outcomes stated with observable verbs ("can compute", "can construct", "can justify"), following the revised Bloom taxonomy [1]. Outcomes are written first; activities and assessment are then chosen to match them [3; 20].

2. **Content component.** The curriculum is split into 16 sections. Inside each section a theoretical lesson introduces definitions, theorems and formulas; one to three practical lessons apply them at increasing difficulty; an assessment lesson closes a block of sections. This rhythm follows the van Hiele idea of moving from visual recognition to deduction [19].

3. **Procedural component — the technological map.** The map is a two-page document: page one holds the goals, outcomes, methods, form of work, assessment criteria, resources and a table of lesson stages; page two describes the course of the lesson step by step. It exists in three variants (Table 1).

4. **Technological component.** Each stage of the map has an assigned digital tool. Dynamic geometry (GeoGebra Classic for plane geometry, GeoGebra 3D for solids) is used at the stages of explanation and worked examples, where a dragged figure shows what stays invariant [6; 10]. Online quiz tools are used at the activation and consolidation stages, where fast feedback matters. Shared documents carry homework and differentiated tasks.

5. **Evaluative component.** Formative assessment runs through every lesson (mini-tests, “Boomerang”, peer checking); summative assessment takes place in 16 control lessons with two parallel variants of five problems, 2 points each, 10 points in total. Results feed back into the plan of the next section.

Table 1. Stages of the 80-minute lesson in the three variants of the technological map

Stage	Theoretical lesson	Practical lesson	Assessment lesson	Digital tool at this stage
Organization	5 min	5 min	5 min	—
Activation	10 min: questions on previous topic, homework check	10 min: formula dictation	15 min: review of key formulas	Online quiz (3–5 items)
Core	30 min: new definitions, theorems, examples	15 min: one worked example	45 min: written test, two variants	GeoGebra construction or 3D model
Application	25 min: tasks in groups	40 min: four graded problems	10 min: analysis of typical errors	GeoGebra files for checking answers
Consolidation and assessment	5 min mini-test, 3 min grading	5 min mini-test, 3 min grading	3 min grading	Online quiz; shared grade sheet
Homework	2 min	2 min	2 min	Shared document with differentiated tasks

The map also records two levels of tasks for differentiation: harder (level C) problems for advanced students and problems close to the worked example for students who need support.

This links the model with the differentiated approach described in [12].

4. Results: application to the two-year curriculum

The model was applied to the calendar plan of the geometry course of an academic lyceum: 144 lessons of 2 academic hours, 288 hours in total, built on the textbooks of J. Husanov and co-authors [7; 8]. Every lesson received a technological map of exactly two pages, so that a map prints on one sheet, front and back. Table 2 shows how the lessons are distributed and which digital tool carries the main visual load in each section.

Table 2. Geometry course of the academic lyceum: sections, lesson types and leading digital tool

No.	Section	Hours	Theoretical	Practical	Assessment	Leading digital tool
1	Axioms of geometry	4	1	1	0	GeoGebra: angles formed by a transversal
2	Geometry of triangles	42	7	12	2	GeoGebra: medians, bisectors, centres of a triangle
3	Compass and straightedge constructions	8	2	1	1	GeoGebra construction protocol
4	Quadrilaterals	20	4	5	1	GeoGebra: Varignon parallelogram
5	Circle and disc	24	4	7	1	GeoGebra: inscribed angles, tangents
6	Polygons	20	4	5	1	GeoGebra: regular polygons, inscribed circles
7	Coordinates and vectors in the plane	20	4	5	1	GeoGebra algebra view
8	Geometric transformations	6	1	1	1	GeoGebra: reflection, rotation, homothety tools
9	Axioms of solid geometry; lines and planes	18	4	4	1	GeoGebra 3D: cube, perpendicular and oblique
10	Coordinates and vectors in space	12	2	4	0	GeoGebra 3D: points, planes, vectors
11	Polyhedra and their surfaces	22	5	5	1	GeoGebra 3D: sections and nets
12	Solids of revolution and their surfaces	16	4	3	1	GeoGebra 3D: surface of revolution
13	Volumes of polyhedra	20	4	4	2	GeoGebra 3D: sliders for dimensions
14	Volumes of solids of revolution	16	3	4	1	GeoGebra 3D: cone and cylinder by slider

No.	Section	Hours	Theoretical	Practical	Assessment	Leading digital tool
15	Combinations of solids	24	5	6	1	GeoGebra 3D: inscribed and circumscribed spheres
16	General revision	16	0	7	1	Online quiz banks
	Total	**288**	**54**	**74**	**16**	

Note: the calendar plan labels 90 lessons as practical; 16 of them are assessment lessons, shown separately.

Along with the maps, a bank of tasks was compiled, each with a checked answer: 108 examples and 162 group tasks for theoretical lessons, 74 worked examples and 296 graded problems for practical lessons, and 160 test items (two variants of five problems) for assessment lessons — 800 items in total. Answers were checked by recomputation; problems with ambiguous or impossible data were rewritten.

Three examples show how a stage of the map is tied to a tool:

- **Lesson 41, mutual position of two circles.** At the core stage students drag the centre of one circle in GeoGebra while a text field shows the distance d between centres and R + r, |R − r|. The five cases (external, touching externally, intersecting, touching internally, nested) appear as the drag passes the two critical values, before the inequalities are written on the board.
- **Lesson 75, theorem of three perpendiculars.** In GeoGebra 3D the teacher builds a plane, a point outside it, its perpendicular and an oblique. Students rotate the scene and see that the projection and the oblique are perpendicular to the same line in the plane. The proof is then written along the rotated picture.
- **Lesson 117, volumes of cylinder and cone.** Sliders for R and h change the two solids with a common base. The ratio of their volumes stays 3 : 1 for any values, and students state the invariant before the formula is introduced.

5. Discussion

The model answers the warning in the dynamic geometry literature that software helps only when its place in the lesson is defined [4; 17]. In the technological map that place is fixed in advance: GeoGebra appears at the core stage of a theoretical lesson and at the worked example of a practical lesson, where a dragged or rotated figure prepares a theorem; quizzes appear at activation and consolidation, where speed of feedback matters more than depth. The teacher does not decide on the spot whether to open the software, and students learn to expect a visual stage in every lesson.

The second gain is alignment. Because outcomes, stages and assessment items are written in one document, the map makes mismatches visible: an outcome such as “can construct a triangle from given elements” with no construction task in the lesson is easy to spot. This is the practical meaning of constructive alignment [3] and backward design [20] at the level of a single lesson.

The two-page limit is a deliberate constraint. It forces the teacher to choose a small number of key concepts and problems for each lesson, and it keeps the map usable in class rather than as a report. The cost is that full lecture notes do not fit in the map; in the proposed model they are a separate appendix, and the map holds only the key formulas and examples.

The model also has limits. First, it depends on equipment: a projector and at least one device per pair of students are needed for the dynamic stages, which not every lyceum classroom has. Second, it depends on teacher knowledge in the TPACK sense [11]: a teacher who has not used GeoGebra 3D will need training before the stereometry sections. Third, and most important, this study is theoretical: it describes the design and its rationale but does not yet measure the effect on achievement.

For this reason a pedagogical experiment is planned. Two groups of 10th-grade students of comparable entry level (an experimental group taught with the maps and digital tools, a control group taught as usual) will be compared over one semester. The instruments will be the 16 assessment lessons of the course, a test of geometric reasoning built on van Hiele levels [19], and a short questionnaire on students' attitude to geometry. Differences will be tested with Student's t-test or the Mann–Whitney U test, depending on the distribution of scores.

6. Conclusion

The article proposed a five-component model for teaching geometry in academic lyceums in which the lesson technological map is the organizing frame and each stage of the map is tied to a digital tool and a form of feedback. The main conclusions are:

1. The technological map and dynamic geometry complement each other: the map defines when and why a digital tool is used; the tool turns static figures into objects of investigation.
2. Three variants of the map — theoretical, practical and assessment — with a fixed 80-minute structure cover all lesson types of the lyceum geometry course.
3. The model is applicable to a complete curriculum: maps were built for all 144 lessons (288 hours, 16 sections) of a two-year course, together with a bank of 800 tasks and test items with verified answers.
4. The two-page format keeps the map a working classroom document and supports alignment of outcomes, activities and assessment.

Recommendations. Lyceum methodological councils can adopt a common map format for mathematics; teachers should receive short practical training in GeoGebra 3D before the stereometry part of the course; maps, GeoGebra files and quiz banks should be stored in a shared repository so that colleagues can reuse and improve them. The next step of the research is the pedagogical experiment described in Section 5.

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