

AN IMPROVED PEDAGOGICAL APPROACH TO TEACHING THE THEORY OF
ELEMENTARY FUNCTIONS

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Abstract. This article recommends teaching the elements of mathematical analysis in schools as the theory of elementary functions. It proposes changing the traditional sequence of topics — “Limit of a sequence, limit of a function, derivative, and integral” — to the following order based on the properties of elementary functions: “Limit of a sequence, limit, derivative, and integral of a continuous function.” The simplification of the traditional approach was achieved by modifying the didactic axiom confirming the continuity of elementary functions and, consequently, adapting the conditions in Heine’s definition of function continuity. In our opinion, this pedagogical approach helps to reduce theoretical gaps and eliminate the difficulties inherent in traditional teaching methods. This work can be considered a simplified interpretation of Academician A.N. Kolmogorov’s idea that the theory of continuous functions should be taught at school, while the theory of general functions should be studied in higher education. We believe that the proposed project can be effectively applied in developing curricula for both general and specialized schools.

Keywords: Continuous line and continuous function, didactic axiom, Heine’s definition, methods of calculating limits, derivatives, and integrals, School mathematics, didactic approaches, methodology of teaching elementary functions.

УСОВЕРШЕНСТВОВАННЫЙ ПЕДАГОГИЧЕСКИЙ ПОДХОД К
ПРЕПОДАВАНИЮ ТЕОРИИ ЭЛЕМЕНТАРНЫХ ФУНКЦИЙ

Аннотация. В статье предлагается преподавать элементы математического анализа в школе как теорию элементарных функций. Предлагается изменить традиционную последовательность тем — «Предел последовательности, предел функции, производная, интеграл» — на следующую, основанную на свойствах элементарных функций: «Предел последовательности, предел, производная, интеграл непрерывной функции». Упрощение традиционного подхода было достигнуто за счёт модификации дидактической аксиомы, подтверждающей непрерывность элементарных функций, и, как следствие, адаптации условий определения непрерывности функций Гейне. По нашему мнению, такой педагогический подход способствует сокращению теоретических пробелов и устранению трудностей, присущих традиционным методам обучения. Данную работу можно считать упрощённой интерпретацией идеи академика А.Н. Колмогорова о том, что теорию непрерывных функций следует преподавать в школе, а теорию общих функций — в вузах. Мы полагаем, что предлагаемый проект может быть эффективно использован при разработке учебных программ как для общеобразовательных, так и для специализированных школ.

Ключевые слова: Непрерывная линия и непрерывная функция, дидактическая аксиома, определение Гейне, методы вычисления пределов, производных и интегралов, школьная математика, дидактические подходы, методика обучения элементарным функциям.

Introduction

At present, it has become a necessity of our time that every highly qualified specialist possess knowledge, skills, and competencies in the most essential branches of mathematics related to their professional field. Consequently, certain concepts and principles of higher mathematics are now being introduced in the upper grades of secondary schools. In particular, topics from the elements of mathematical analysis — such as limits, derivatives, and integrals — are included among these.

The foundation for studying these topics is established through the concept of the limit of a function. In this article, we propose and justify a methodologically new approach to this issue by analyzing how elements of mathematical analysis are taught in the schools of developed countries.

Based on the historical development of mathematics, we consider it appropriate to focus only on problems related to the theory of elementary functions (limits, derivatives, and integrals) in school, and to continue this study logically in higher education. To achieve this, we introduce a special definition of limits for elementary functions.

In this regard, some countries have adopted either the Cauchy or Heine definition of function limits as their primary approach, and have selected the following sequence of topics: Limits of sequences → Limits of functions → Derivatives → Integrals.

The methods presented in the aforementioned literature focus primarily on developing students' practical skills. However, due to the large volume and complexity of the material and limited time, the theoretical content is often presented without sufficient justification. In addition, not enough attention is paid to developing creative thinking. As a result, teaching function theory in higher education often has to start from scratch, and the expected outcomes may not be achieved.

In our opinion, considering the theoretical gaps in the current system and the limited number of teaching hours, it is possible to address these issues by implementing the study of elementary function theory in schools. We believe that teaching the general theory of functions in higher education based on the theory of elementary functions will enhance students' creative thinking abilities and produce higher quality future specialists.

Why should this approach be adopted? Looking at the history of mathematics, we see that the concept of a line predates the concepts of function and its graph. The graph of a function is merely one representation of a line. A continuous line is a fundamental concept, and the graphs of elementary functions consist of continuous lines. Taking this into account, we introduce the continuity of elementary function graphs in the results section of our article through a didactic axiom. Unlike the traditional method, this approach introduces the continuity of a function without relying on the concept of limits. Building on this, we then introduce the concept of limits as it pertains to elementary functions. Consequently, it becomes easier to justify many theoretical considerations.

Methods

Our results depend on the methods described above for finding the limit of a sequence and the limit of a function using Heine's definition. According to Heine's definition, the function $f(x)$ has a finite limit L as $x \rightarrow a$ if the following holds: "For any sequence (x_n) converging to the number a , where $(x_n \neq a)$, if the sequence $f(x_n)$ converges to some number L , then this number is called the limit of $f(x)$ as $x \rightarrow a$, and is written as: $\lim_{x \rightarrow a} f(x) = L$ ".

In the special case $L = f(a)$, the function is called continuous at the point $x = a$.

In the definitions of Heine or Cauchy, the continuity of a function at a point or on a set is expressed by the concept of a limit. If we consider the graph of elementary functions as a continuous line, then continuity can be introduced as an intuitive initial concept using the axiomatic method. In other words, we can claim the continuity of elementary functions in the domain of definition without the concept of a limit, and we do not contradict this.

Let the graph of the function be a continuous line. Let us represent on the graph of the function some sequence (x_n) and the corresponding sequence $(f(x_n))$, which tends to a . In this case, we obtain points (n, x_n) corresponding to $(n; f(x_n))$. In $n \rightarrow \infty$, these points tend to the point $(\infty; f(a))$ on the graph of the function, that is, the relation $\lim_{n \rightarrow \infty} f(x_n) = f(a)$ holds. Similarly, any sequence $(y_n): y_n \rightarrow a, n \rightarrow \infty$ different from (x_n) and the corresponding sequence of points on the graph $(n; f(y_n))$ in $n \rightarrow \infty$ will also be $(\infty, f(a))$, that is $\lim_{n \rightarrow \infty} f(y_n) = f(a)$ will be true. From this, the following conclusion can be drawn. When $f(x)$ is continuous, the expression "for any sequence $(x_n) (x_n \neq a)$ " in Heine's definition of function continuity at $x = a$ can be replaced with "for some sequence (x_n) with $(x_n \neq a)$, that tends to a " that tends to a .

Results

The object of our research is the class of elementary functions (polynomials, trigonometric, exponential, logarithmic functions), and our aim is to describe their differential and integral calculus.

It is known that a line is one of the fundamental concepts in geometry. The graph of a function is a special type of line, the points of which are determined by a specific formula. A continuous function is a line whose graph can be drawn over a certain interval without lifting the pen, i.e., it is a line without breaks or discontinuities.

Based on the above, we present our didactic axiom that affirms the continuity of elementary functions without proof.

Definition 1: If for some sequence $(x_n) (x_n \neq a)$, approaching a , the sequence $(f(x_n))$ approaches the number $f(a)$, this number is called the limit of $f(x)$ as $x \rightarrow a$ and is written as $\lim_{x \rightarrow a} f(x) = f(a)$

$\lim_{x \rightarrow 2} x^2$ For example, suppose we need to determine. By definition

$x_n = 2 + \frac{1}{n} \rightarrow 2, n \rightarrow \infty$, we take a sequence and

$f(x_n) = (2 + \frac{1}{n})^2 = 4 + \frac{2}{n} + \frac{1}{n^2}, n = 1, 2, \dots$, We determine the limit of the sequence $f(x_n)$.

$\lim_{n \rightarrow \infty} f(x_n) = \lim_{n \rightarrow \infty} (4 + \frac{2}{n} + \frac{1}{n^2}) = 4 + 0 + 0 = 4$ $\lim_{x \rightarrow 2} x^2$ For example, suppose we need to determine. By definition

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$$\lim_{x \rightarrow 1} \arcsin x = ? \quad x_n = 1 - \frac{1}{n} \rightarrow 1, n \rightarrow \infty \quad \lim_{n \rightarrow \infty} f(x_n) = \lim_{n \rightarrow \infty} \arcsin(1 - \frac{1}{n}) = \arcsin 1 = \frac{\pi}{2}$$

For example, There fore. $\lim_{x \rightarrow 1} \arcsin x = \frac{\pi}{2}$

Note According to the traditional Heine definition, our conclusion in this case would be incorrect, because we did not consider an arbitrary sequence.

If $a \notin D(f)$ and $L \in \mathbb{R}$ is some finite number, then the limit of the function on $x \rightarrow a$ is defined as follows.

Definition 2 : If for some sequence (x_n) approaching a , the sequence $(f(x_n))$ tends to a number L , then this number is called the limit of $f(x)$ as $x \rightarrow a$, and it is written as $\lim_{x \rightarrow a} f(x) = L$.

Similar definitions can be formulated for the cases when $= \pm\infty$, and when L is either finite or infinite. Now, based on the axioms and definitions, we will derive the definition of the derivative of a function at a point x_0 .

A new pedagogical approach to teaching the theory of elementary functions at the school level is implemented through solving the following problems based on established axioms and definitions:

- *Solving problems involving the indefinite integrals of elementary functions, evaluating definite integrals, and applying the Newton-Leibniz formula.*

Certainly, solving such problems is relatively easier compared to the general theoretical framework. If students pursue higher education later on, it will not be difficult to explain these concepts and theorems to them in a generalized and comprehensive manner.

Discussion

The purpose of this study was to substantiate a new pedagogical approach to teaching differential and integral calculus within the theory of elementary functions in schools. To achieve this goal, two theoretical tools were proposed and presented in the results section.

The first of these is a didactic axiom corresponding to elementary functions, according to which we accept—without formal proof—that elementary functions are represented as continuous curves within their domains of definition. This didactic axiom enables school students to treat elementary functions as continuous functions.

With the acceptance of the axiom, a second theoretical instrument emerges. Now, if we present the traditional Heine definition of the limit of a function for a continuous line, i.e., a continuous function, we derive the definition of the limit of elementary functions in terms of sequences. In definitions 1 and 2 in the results section, we now see that the phrase "for any sequence (x_n) approaching a " in Heine's definition has been replaced by "for some sequence (x_n) approaching a ." "If students pursue higher education, encountering Heine's general definition will not come as a surprise to them". In particular, they can understand the derivation of definitions 1 and 2. The scheme for studying function theory in higher education is as follows: "Sequence limit $\rightarrow$ function limit $\rightarrow$ function continuity $\rightarrow$ function derivative $\rightarrow$ function integral," while in elementary function theory, this scheme becomes: "Sequence limit $\rightarrow$ continuous function limit $\rightarrow$ function derivative $\rightarrow$ function integral." These topics are within the scope of elementary functions, briefly stating theoretical and practical results only for elementary functions. In higher education, issues of generalization are considered. This, of course, helps to enhance the student's creative abilities.

The idea we introduced is not new. The concept of teaching only the theory of continuous functions in school and the theory of general functions in higher education was proposed by Academician A.N. Kolmogorov. An attempt to implement this idea in a strong form was made by Y.N. Vilenkin and A.G. Martkovich in the literature [14]. In it, the teaching of elementary functions in the form of "Sequence limit $\rightarrow$ function limit $\rightarrow$ function continuity $\rightarrow$ function derivative $\rightarrow$ function integral" remains.

The continuity of elementary functions is proven using Cauchy's definition, and the main instrument is defined as the limit of a continuous function: $\lim_{x \rightarrow a} f(x) = f(a)$.

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